

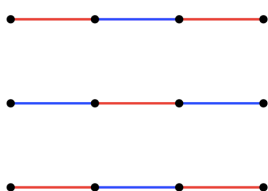
Dots & Boxes
A Combinatorial Game Analysis
Expository paper

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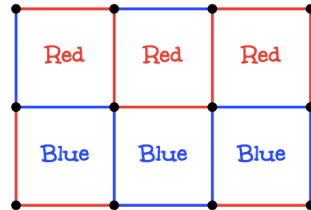
Dots and Boxes is a short combinatorial game. For a game to be combinatorial it must adhere to the following three rules: a game must be sequential, players of a game must have perfect information, and a game must not be one of chance. Sequential games are ones in which players alternate turns, Rock-Paper-Scissors is not sequential since players move simultaneously. Players have perfect information if they know the entire state of the game. This information includes all the previous moves and all possible moves of any player. A game with no chance has no randomization involved (e.g., dice or shuffling). Lastly, a short game is one in which there exists a finite number of reachable positions. Hence, no positions can be repeated.

The game board consists of a rectangular grid of dots orthogonal to one another. Gameplay involves two players taking turns moving a minimum of once. A move is initiated with the drawing of a connecting line between two adjacent dots. If a player's move closes a square box, the player claims that box with their initials and must move again if possible. The winning player is the one with the most boxes claimed after no more moves can be made. There are multiple strategies that can either help and hurt the player that opts to use them. Principles and strategies I will be discussing include: the greedy strategy, long chains, double-dealing, and parity of the game.

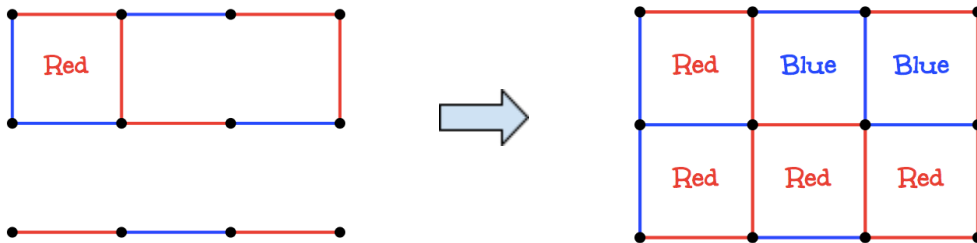
A greedy strategy is one in which a player “always chooses the move that maximizes or minimizes some quantity related to the game position” [1]. In our particular game, this would be claiming as many boxes as one can at any given opportunity. Although a player wants to have the most amount of boxes by the end of the game, this strategy can often cause a loss for the one that employs it. In some cases the optimal play is to refrain from claiming available boxes if it allows you to, in future moves, claim a larger total amount. For example, as shown below, we have a game with two rows with three or more adjacent boxes, this is called a long chain [2].



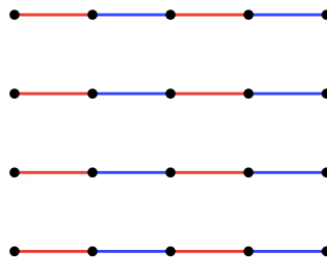
Suppose it is your turn and you employ the greedy strategy.



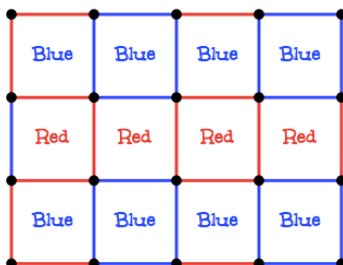
The game ends in a tie. Now let's claim the first box but sacrifice the remaining two in the top most long chain, this move is referred to as double-dealing.



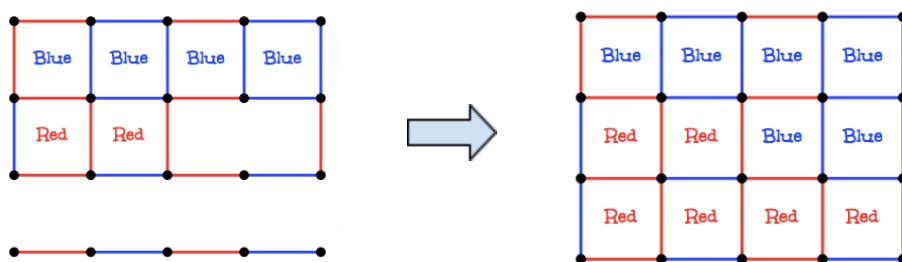
It became a 4-2 win for Red. This strategy has to do with changing the parity of a game. Parity describes whether a quantity is even or odd. For a quick example, the parity of 2 is even while the parity of 3 is odd. The parity of the remaining unclaimed boxes can be very important to take note of, as it can be helpful in forecasting the outcome of a game. Double-dealing is one way to change the parity of a game. An example of this is as follows,



It is your turn in the above position. Employing the greedy strategy, the first player to move, Red, will always lose because there exists an odd number of rows. This is shown below.



However, now that you know to use double-dealing to change the parity, you are able to force a draw.



With these strategies in mind, you should now be able to win against an amateur opponent.

Works Cited

- [1] ALBERT, MICHAEL H, et al. LESSONS IN PLAY: an Introduction to Combinatorial Game Theory. Taylor & Francis Group, 2007.

- [2] Rhoads, Glenn. "The Strategy of Dots and Boxes." gcrhoads.byethost4.com, gcrhoads.byethost4.com/DotsBoxes/dots_strategy.html?i=1.

- [3] Manners, Ossie. "Dots And Boxes." Dots And Boxes, dotsandboxes.org/.